Supplementary Problems

5.46 Use the free body diagram method to derive the differential equations governing the motion of the system of Fig. 5-27 using x_1, x_2, and x_3 as generalized coordinates.

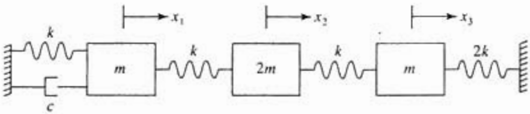

Fig. 5-27

Ans.

$$m\ddot{x}_1 + c\dot{x}_1 + 2kx_1 - kx_2 = 0$$
$$2m\ddot{x}_2 - kx_1 + 2kx_2 - kx_3 = 0$$
$$m\ddot{x}_3 - kx_2 + 3kx_3 = 0$$

5.47 Use the free body diagram method to derive the differential equations governing the motion of the system of Fig. 5-28 using θ_1 and θ_2 as generalized coordinates.

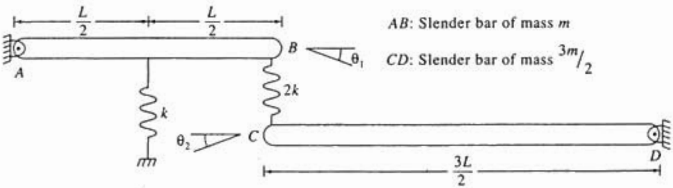

Fig. 5-28

Ans.

$$\tfrac{1}{3}mL^2\ddot{\theta}_1 + \tfrac{9}{4}kL^2\theta_1 - 3kL^2\theta_2 = 0$$
$$\tfrac{9}{8}mL^2\ddot{\theta}_2 - 3kL^2\theta_1 + \tfrac{9}{2}kL^2\theta_2 = 0$$

5.48 Use the free body diagram method to derive the differential equations governing the motion of the system of Fig. 5-29 using θ, x_1, and x_2 as generalized coordinates.

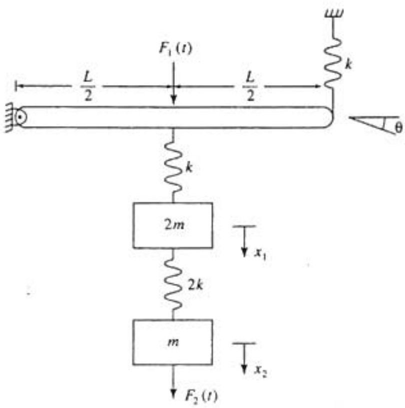

Fig. 5-29

Ans.

$$\tfrac{1}{3}mL^2\ddot{\theta} + \tfrac{5}{4}kL^2\theta - \tfrac{1}{2}kLx_1 = \tfrac{1}{2}F_1(t)L$$
$$2m\ddot{x}_1 - \tfrac{1}{2}kL\theta + 3kx_1 - 2kx_2 = 0$$
$$m\ddot{x}_2 - 2kx_1 + 2kx_2 = F_2(t)$$

5.49 Use Lagrange's equations to derive the differential equations governing the motion of the system of Fig. 5-27 using x_1, x_2, and x_3 as generalized coordinates.

Ans.

$$\begin{bmatrix} m & 0 & 0 \\ 0 & 2m & 0 \\ 0 & 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} c & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 3k \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

5.50 Use Lagrange's equations to derive the differential equations governing the motion of the system of Fig. 5-28 using θ_1 and θ_2 as generalized coordinates. Assume small θ, and write the differential equations in matrix form.

Ans.

$$\begin{bmatrix} \tfrac{1}{3}mL^2 & 0 \\ 0 & \tfrac{9}{8}mL^2 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} \tfrac{9}{4}kL^2 & -3kL^2 \\ -3kL^2 & \tfrac{9}{2}kL^2 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

5.51 Use Lagrange's equations to derive the differential equations governing the motion of the system of Fig. 5-29 using θ, x_1, and x_2 as generalized coordinates.

Ans.

$$\begin{bmatrix} \frac{1}{3}mL^2 & 0 & 0 \\ 0 & 2m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} \frac{5}{4}kL^2 & -\frac{1}{2}kL & 0 \\ -\frac{1}{2}kL & 3k & -2k \\ 0 & -2k & 2k \end{bmatrix} \begin{bmatrix} \theta \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}F_1(t)L \\ 0 \\ F_2(t) \end{bmatrix}$$

5.52 Use Lagrange's equations to derive the differential equations governing the motion of the system of Fig. 5-30 using θ_1 and θ_2 as generalized coordinates. Assume small θ_1 and θ_2.

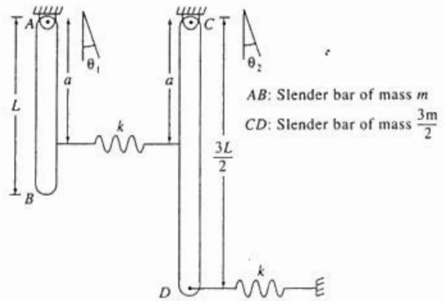

Fig. 5-30

Ans.

$$\begin{bmatrix} \frac{1}{3}mL^2 & 0 \\ 0 & \frac{9}{8}mL^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} ka^2 + \frac{1}{2}mgL & -ka^2 \\ -ka^2 & k(a^2 + \frac{9}{4}L^2) + \frac{3}{4}mgL \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

5.53 Use Lagrange's equations to derive the differential equations governing the motion of the system of Fig. 5-31 using x and θ as generalized coordinates. Assume small θ.

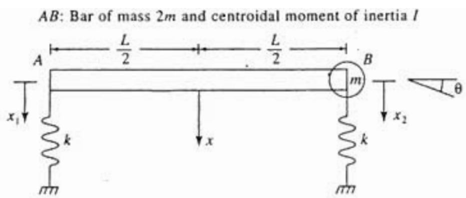

Fig. 5-31

Ans.

$$\begin{bmatrix} 3m & \frac{1}{2}mL \\ \frac{1}{2}mL & I + \frac{1}{4}mL^2 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} 2k & 0 \\ 0 & \frac{1}{2}kL^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

5.54 Use Lagrange's equations to derive the differential equations governing the motion of the system of Fig. 5-31 using x_1 and x_2 as generalized coordinates.

Ans.

$$\begin{bmatrix} \frac{m}{2}+\frac{I}{L^2} & \frac{m}{2}-\frac{I}{L^2} \\ \frac{m}{2}-\frac{I}{L^2} & \frac{3}{2}m+\frac{I}{L^2} \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}+\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

5.55 Use Lagrange's equations to derive the motion of the system of Fig. 5-32 using x_1, x_2, and θ as generalized coordinates.

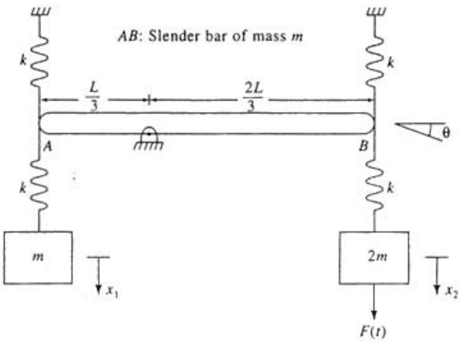

Fig. 5-32

Ans.

$$\begin{bmatrix} \frac{1}{9}mL^2 & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 2m \end{bmatrix}\begin{bmatrix} \ddot{\theta} \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}+\begin{bmatrix} \frac{10}{9}kL^2 & \frac{1}{3}kL & -\frac{2}{3}kL \\ \frac{1}{3}kL & k & 0 \\ -\frac{2}{3}kL & 0 & k \end{bmatrix}\begin{bmatrix} \theta \\ x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ F(t) \end{bmatrix}$$

5.56 Use Lagrange's equations to derive the differential equations governing the motion of the system of Fig. 5-33 using x_1, and x_2 as generalized coordinates. Assume the disk rolls without slip.

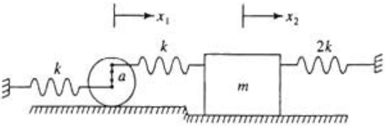

Fig. 5-33

Ans.

$$\begin{bmatrix} \frac{3}{2}m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k\left(2+2\frac{a}{r}+\frac{a^2}{r^2}\right) & -k\left(1+\frac{a}{r}\right) \\ -k\left(1+\frac{a}{r}\right) & 3k \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

5.57 Use Lagrange's equations to derive the differential equations governing the motion of the system of Fig. 5-34 using x_1, x_2, and θ as generalized coordinates.

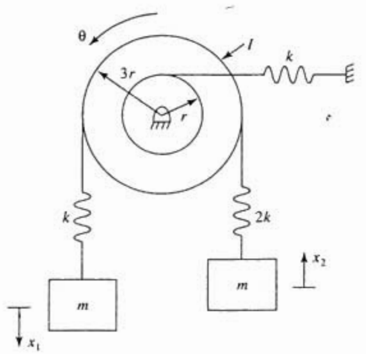

Fig. 5-34

Ans.

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} k & 0 & -3kr \\ 0 & 2k & -6kr \\ -3kr & -6kr & 28kr^2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

5.58 Derive the differential equations governing the motion of the system of Fig. 5-35 using θ_1 and θ_2, as generalized coordinates.

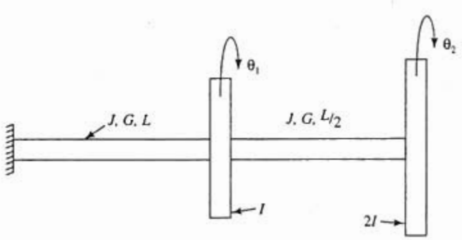

Fig. 5-35

Ans.

$$\begin{bmatrix} I & 0 \\ 0 & 2I \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \frac{JG}{L}\begin{bmatrix} 3 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

5.59 Derive the differential equations governing the motion of the system of Fig. 5-36 using θ_1, θ_2, and x as generalized coordinates.

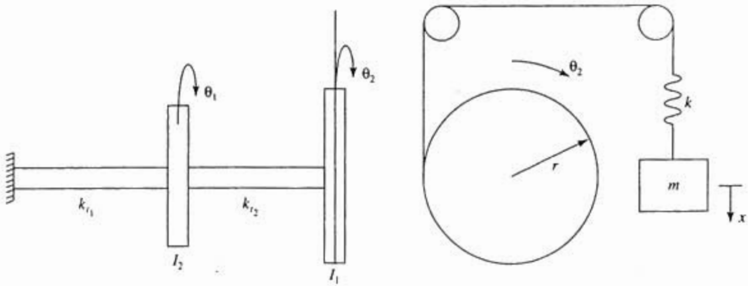

Fig. 5-36

Ans.

$$\begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & m \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} k_{t_1} + k_{t_2} & -k_{t_1} & 0 \\ -k_{t_1} & k_{t_2} + kr^2 & -kr \\ 0 & -kr & k \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

5.92 Determine the general free vibration response of the system of Fig. 5-40.

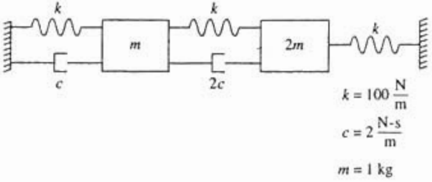

Fig. 5-40

Ans.

$$e^{-0.268t}\left\{C_1\begin{bmatrix}0.732\\1\end{bmatrix}\cos 7.96t + C_2\begin{bmatrix}0.732\\1\end{bmatrix}\sin 7.96t\right\}$$

$$+\,e^{-3.73t}\left\{C_3\begin{bmatrix}-2.73\\1\end{bmatrix}\cos 14.92t + C_4\begin{bmatrix}-2.73\\1\end{bmatrix}\sin 14.92t\right\}$$